RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2011

FIRST YEAR

PHYSICS (Honours)

Date : 16/12/2011

Time : 11am – 2pm

Paper : I

Full Marks : 75

[Use separate answer-books for each group]

Group–A

Answer **any five** questions out of 8 questions: 10x5

1. a) Show that the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$ is convergent but not absolutely convergent. 4

   b) Evaluate the double integral $I=\iint_R e^{-(x^2+y^2)}dxdy$, where R is the region bounded by the circle $x^2+y^2=a^2$, by transforming to polar coordinates. 3

   c) For the Gaussian (or normal) distribution, $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, show that the mean is μ, and variance is σ^2. 3

2. a) Prove that a necessary and sufficient condition that $\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\times\vec{B})\times\vec{C}$, is that $(\vec{A}\times\vec{C})\times\vec{B}=\vec{0}$. 4

   b) If $\vec{a}_1=(1,1,0),\ \vec{a}_2=(0,1,0), \vec{a}_3=(1,1,1)$, find their reciprocal set of vector $\vec{b}_1,\vec{b}_2,\vec{b}_3$. Verify that $\vec{a}_1\cdot\vec{a}_2\times\vec{a}_3=\frac{1}{\vec{b}_1\cdot\vec{b}_2\times\vec{b}_3}$. 3

   c) Show that under the coordinate transformation

   $$x'=x\cos\theta+y\sin\theta$$
   $$y'=-x\sin\theta+y\cos\theta$$
   $$z'=z$$

   the scalar product $\vec{A}\cdot\vec{B}$ of two vectors is invariant. 3

3. a) (i) Define directional derivative of a scalar point function $\phi(x,y,z)$, in the direction of the vector $\vec{a}$. 2

   (ii) Given $\phi=xy+yz+zx$, find $\nabla\phi$ at (1, 1, 3) and directional derivative of ϕ at (1, 1, 3) in the direction of the vector (1, 1, 1). 2

   b) Show that by integrating along a closed curve c in the x–y plane

   $$\left|\oint_c \vec{r}\times d\vec{r}\right|=2A,$$

   where A is the area bounded by c. 3

   c) (i) If $f(r)$ is a scalar function of r, where $r=|\vec{r}|$, evaluate $\nabla^2 f(r)$.

(ii) If $\nabla^2 f(r) = 0$, find the nature of $f(r)$. $(r \neq 0)$ 3

4. a) Define a regular singular point of an ordinary differential equations. Use Frobenius method to find the general solutions of the differential equation $xy'' + 2y' + xy = 0$. 5

   b) The generating function for Hermite polynomial is $\exp(-s^2 + 2sz) = \sum_{n=0}^{\infty} \frac{s^n}{n!} H_n(z)$, where $H_n(z)$ is nth order Hermite polynomial. Prove that (i) $\frac{dH_n}{dz} = 2nH_{n-1}$, (ii) $2ZH_n = H_{n+1} + 2nH_{n-1}$. 5

5. a) Consider the sawtooth function $f(x)$, given by

$$f(x) = x+1, \quad -1 \le x \le 0$$
$$= -x+1, \quad 0 \le x \le 1$$
$$f(x+2) = f(x)$$

   (i) Plot the function.

   (ii) Expand in Fourier Series.

   (iii) Hence show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$. 1+4+2

   b) Show that for the Dirac δ-function below,

$$\delta(x^2 - a^2) = \frac{1}{2|a|}\{\delta(x-a) + \delta(x+a)\}$$ 3

6. a) An unitary matrix U is written as $U = A + iB$, where A & B are Hermitian. Show that $AB = BA$.

   b) Consider a 2x2 unitary matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the modulus of whose determinant is unity. Find the relations obeyed by a, b, c, d. Hence find the no. of independent parameters to specify such matrix.

   c) A rotation is given by an orthogonal matrix. Show that the matrix

$$\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

   describes a rotation. What is the axis of rotation in this case? 2+4+4

7. Consider a region in plane bounded by unit circle $x^2+y^2 = 1$. The top half and the bottom half of the boundary is maintained at potentials V & $-V$ respectively. Write down 2–dim. Laplace's equation in Catesian coordinate and transform the equation in polar coordinates. Solve the resulting equation with the above boundary condition for potential at an inside point. (You do not have to evaluate any integral explicitly). 10

8. a) Consider the one–dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, $y \equiv y(x,t)$. Show that $y_1 = f(x+ct)$ and $y_2 = g(x-ct)$ are both solutions of the equation, hence write down the general solution. 4

b) Solve the two–dimensional Laplace equation for $T(x, y)$ with the boundary conditions:

(i) $T(0, y) = T(l, y) = 0, \quad 0 < y < a$

(ii) $T(x, 0) = 0, \quad 0 < x < l$

(iii) $T(x, a) = 100$

(The coefficients of expansion in the general solution need not be evaluated). 6

Group – B

Answer **any two** questions: 10x2

9. a) State and explain Fermat's principle. Derive the laws of refraction at a curved surface from Fermat's principle. 1+2+4

b) Consider a refracting surface of radius of curvature '*R*', separating two media of indices n_1 and n_2. A straight line passing through the centre of curvature *C* intersects the refracting surface at *O*. *P* and *Q* are two points on the straight line on opposite sides of the refracting surface such that *PO* = *OQ* = *x*. Show that the straight line path *POQ* corresponds to an extremum. 3

10. a) Obtain the system matrix for a thick lens and hence obtain the system matrix for a thin lens. 2+2

b) Find by matrix method the relation $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, in case of refraction at a curved surface, the symbols having their usual significance. 4

c) Show that the planar surface of a concave planar or convex planar lens does not contribute to the system matrix. 2

11. a) Define linear magnification and angular magnification of an optical system. Derive Helmholtz-Lagrange relation between these two types of magnification (using matrix method). 2+4

b) What do you mean by dispersive power of a material? Deduce the condition of achromatism of two lenses, of same material, separated by a distance. 1+3

12. a) What is spherical aberration? How can it be removed using two convex lenses separated by a distance? 1+3

b) What is meant by aplanatic foci of a lens system? Find the position of aplanatic foci of a spherical refracting surface. Give its practical application. 1+3+2

Answer question no. 13 or 14: 5x1

13. a) Show that a single lens of finite focal length can not be made achromatic. 2

b) What is an eye piece? Why should it consist of two lenses? 3

14. a) What are the 'Principle of independence' and the 'Principle of reversibility' of light paths. 3

b) Light takes 0.02 sec to move over the distance between two points *A*, *B* in vacuum. The space is next filled with a liquid of index 1.33. What now will be the time taken by light to travel between the same two points? 2

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